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Engineering Notes

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Optimal Sensor Placement of Large Flexible Space Structures

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Introduction

THIS Engineering Note describes an experimental evaluation of a predeveloped sensor placement algorithm derived from the D-optimality criteria for experiment design on a flexible planar truss. The D-optimality criteria, developed by Bayard et al.[1] for optimal experiment design for identification of large space structures, is defined as the maximum determinant of the Fisher information matrix. It results in a separation principle that decouples the problems of optimal input design and optimal sensor design such that each can be solved independently of each other. The sensor placement design is experimentally verified on a 63-degree-of-freedom planar truss. The results are presented in this Note. Although there are many different sensor and actuator methods in the literature, few have been applied experimentally to a physical system. The sensor placement algorithm is implemented and programmed using data relevant to the planar truss. Various sensor locations, as determined by the algorithm, are tested to determine how well the sensors are able to identify the natural frequencies associated with the planar truss.

Experimental Flexible Structure

In the University of Washington Control Systems Laboratory (UWCSL), a 23.2-ft planar truss structure modeled after the truss at the U.S. Air Force Academy, Colorado Springs, Colorado,[2] has been built for research and educational purposes. The 239-lb truss has 20 bays, lies flat in the horizontal plane, and is supported by low rolling friction, $\frac{3}{4}$-in. steel balls. The diagonal and side dimensions of each square bay are 19.69 and 13.90 in., respectively. One end is free, whereas the other end is bolted to a rigid steel table, which itself is anchored into the concrete floor of the laboratory. The longitudinal, diagonal, and chordwise members are constructed from aluminum and are connected with threaded steel joints. Attached to the 20 chordwise truss members (not counting the constrained end location) are rigid steel bars to increase the structure's stiffness and mass and to prevent motion in the upward direction. The ends of the individual truss members are fitted with steel bolt assemblies to provide a rigid connection with the threaded steel joints.[3]

Air jet thrusters (AJTs) are utilized as the primary source to excite and control the lateral motions of the structure. The AJTs are oriented back-to-back to provide a bang-bang type of actuation. Each pair of AJTs is fired transversely in opposite directions by on–off triggered circuits where a positive control signal voltage fires one AJT and a negative control signal voltage fires the opposite AJT. The truss lateral motions are measured by high-precision, low-noise accelerometers.

Separation Principle for Optimal Experiment Design

The D-optimality criteria is defined as the maximum determinant of the Fisher information matrix. The maximization is taken with respect to the experimental design parameters subject to constraints as imposed in the design requirements. A particular result of the D-optimal experiment design is a decoupling effect wherein the optimal input design problem (for identification) and the optimal sensor placement problem can be solved independently. This separation effect on the experiment design is possible because of the light damping characteristics inherent in flexible space structures.[1]

The optimal sensor placement design is presented mathematically as follows:

$$\beta^* = \operatorname{Arg}\max_{\beta} \left\{ \prod_{i=1}^{N} \left(\sum_{k=1}^{M} \beta_k \frac{\left(r_k^T \phi_i\right)^2}{\psi_k^2} \right) \right\} \tag{1}$$

where N is the number of significant structural mode shapes and M is the number of points that describes a particular mode shape. The vector $\boldsymbol{r}_k \in R^M$ is associated with the location of the kth sensor, ϕ_i are the N flexible body mode shape vectors, and ψ_k is the noise spectrum associated with the kth sensor. The vector $\beta_k \in R^M$ is composed of 1s and 0s and has a value of 1 if that particular point in the mode shape is included in the output and 0 otherwise.

It is seen from Eq. (1) that the determination of the optimal sensor placement vector β^* requires only knowledge of the structure's natural mode shapes. It is independent of the values of the modal frequencies and dampings. The mode shapes are easily attainable from one of the numerous finite element modeling software packages available on the market.

For computational purposes, Eq. (1) can be written in another form. By taking the log of both sides, Eq. (1) can be written in the form

$$S^*(m) = \max_{\beta} \sum_{i=1}^{N} \log \left(\sum_{k=1}^{M} \beta_k \frac{\left(r_k^T \phi_i\right)^2}{\psi_k^2} \right) \tag{2}$$

subject to $\beta \in B_m$, where

$$B_m = \left[\beta : \sum_{k=1}^{M} \beta_k = m \right] \tag{3}$$

The number of sensors m is chosen from the total candidate number of sensors M. The total number of sensors is constrained to be equal to m (where $m \leq M$). One can see then that the algorithm provides two different kinds of information. For a given total candidate number of sensors M, the algorithm provides information as to what the optimal number of sensors should be. In addition, for each number of sensors m, the optimal sensor location is also determined.

Sensor Placement Algorithm Applied to UWCSL Truss

The original design of the UWCSL truss uses two sensors positioned at the tip and midpoint (referred to as stations 20 and 10). These locations were chosen from a heuristic standpoint of providing symmetry in the structure. It is shown that a midpoint location

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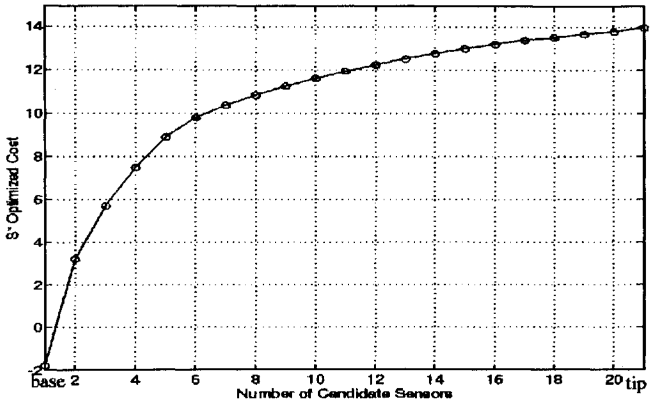

Fig. 1 UWCSL truss: optimized cost vs number of sensors.

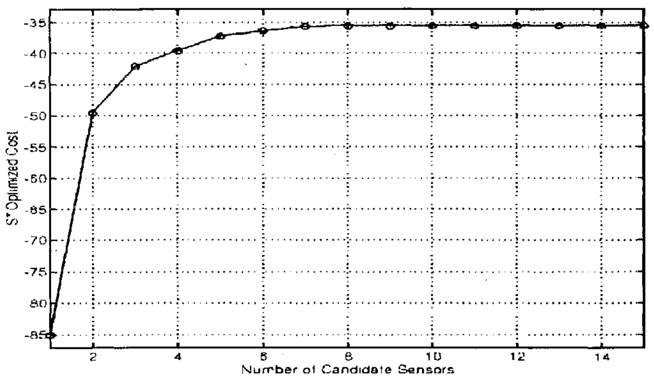

Fig. 2 JPL power tower model, optimized cost vs number of sensors.

is not a good sensor location in general. The placement algorithm is run utilizing the first four bending mode shapes of the truss. These are the significant mode shapes of the truss based on the physical limitation and dynamic contribution of the truss structure. Each mode shape is identified by 21 points.

Figure 1 shows a plot of the optimized cost vs the number of sensors used. Note that the plot does not level off at a certain number of sensors. This leveling would indicate the optimal number of sensors required to identify and detect all possible motions. The Jet Propulsion Laboratory (JPL) simulated study of the Space Station power tower model indicated an optimal number of seven sensors, as shown in Fig. 2.[1] Note also that the cost value computed has no physical interpretation except as a relative measure of optimality between the number of sensors and the actual location for a specified number of sensors.

To generate the results shown in Fig. 2, the placement algorithm was utilized to determine the possible optimal number of sensors given the mode shapes and number of candidate sensors. The algorithm's most advantageous use, however, is the fact that for a given number of sensors the algorithm gives a ranking as to where the sensors should be placed. This is useful in light of the size and weight restrictions typical of space structures. For example, what if only one sensor is available for use? The algorithm not only ranks the optimal location but also provides an alternate location. This is handy if one ends up having a physical restriction on the structure that makes it impossible to place a sensor at that location. Another situation involves having two sensors available but measurements are restricted to only one sensor at a time. In this case, one of the sensors is placed at the optimal location as determined by the algorithm and the second sensor at the second optimal location. The second sensor acts as a backup in case of failure on the first sensor. Such an application is desirable especially if there is a long waiting period for repairs.

Table 1 shows the top five candidate locations for one sensor. The locations are ranked from top to bottom. The cost for each location is also shown. The optimal location for one sensor, as determined by the algorithm, is at the tip (station 20) of the truss. This agrees intuitively as to where a sensor should be located for a simple cantilever-modeled truss.

Table 1 Top locations and corresponding costs: one sensor

Station	Costs
20	5.8546
19	2.3952
14	1.3717
11	0.9903
8	0.5623

Table 2 Top locations and corresponding costs: two sensors

Station	Costs
19 and 20	7.4041
4 and 20	7.2729
15 and 20	7.2001
3 and 20	7.1445
9 and 20	7.1211

For the two-sensor case, Table 2 shows the top five candidate locations. The ranking is from top to bottom. Once again, the tip (station 20) plays a significant role. One can see that in each of the top five pairs of locations, a sensor at station 20 is necessary.

Experimental Results for UWCSL Truss Using One Sensor

Two AJTs mounted back to back are positioned at the tip of the planar truss as a means to excite the different modes of the system. Based on the computational results shown in Table 1, one accelerometer was placed accordingly. Utilizing a dynamic signal analyzer (DSA), a swept sine with a volt peak of 0.05 V was used as the input source. The frequency sweep was from 1 to 51 Hz, which just covers the bandwidth of the AJTs.

A Bode frequency response is generated with the DSA. The natural frequencies of the UWCSL planar truss occur where the peaks are as seen from the magnitude plot. Figure 3 illustrates a sample Bode frequency response as generated by the DSA. The response in Fig. 3 is for the case where the accelerometer is located at the tip position (station 20), which is the optimal location. Note that the finite element model did not take into account the mass/locations of the AJTs and the accelerometers. The masses are negligible compared to the mass of the rigid steel bars and thus should hardly affect the truss' natural frequencies. This is not the general case, however, because actuators are typically more expensive and massive than sensors. Thus, the optimal number and placement of actuators is likely to have a greater significance than sensor optimization. This is especially true for space applications.[4]

Table 3 gives a summary of the natural frequencies and the corresponding gains for the top five optimal locations for one sensor. All five optimal locations were able to identify the natural frequencies associated with the UWCSL truss. In addition to the top five optimal locations, an experimental run was conducted where the measurements were made at the midpoint (station 10). The gain was used as a relative index to determine the effectiveness of the different candidate locations in identifying the natural frequencies of the UWCSL planar truss. A comparison of the different gains clearly gives station 20 superiority over the rest (although station 19 is marginally better in identifying the first frequency). Station 19 is the second optimal location. It is seen that station 19 is almost as good as station 20, except for the slight difficulty in identifying the fourth frequency. This can be attributed to the fact that a node exists very close to that point. Station 14 gives almost equal gain values for the first three frequencies and fares poorly with the fourth frequency. Similar to station 19, a node exists close to that point. Station 11 and station 8 are about equally matched except for a slight superiority of station 11 in identifying the fourth frequency. As shown in Table 3, station 10 is not able to identify the third natural frequency. This is because a node exists at that point for the third mode shape.

Overall, the experimental validation of the sensor placement algorithm can be considered successful. Most importantly, all top five optimal sensor candidate locations (for one sensor) successfully identified all four significant natural frequencies. Knowledge

Table 3 Natural frequencies and gains for different sensor locations

Freq., Hz	Gain, dB	Freq., Hz	Gain, dB	Freq., Hz	Gain, dB
Station 20		*Station 19*		*Station 14*	
1.485	34.2914	1.544	36.0171	1.485	33.8945
9.14	39.1421	9.14	36.8249	9.14	33.8988
24.545	32.7966	24.545	28.918	23.594	34.1208
44.398	17.3231	42.678	10.1763	42.678	12.1352
Station 11		*Station 8*		*Station 10*	
1.544	27.9422	1.544	25.1474	1.544	26.5891
9.14	39.2694	9.14	39.256	9.14	39.615
24.545	24.2948	23.594	27.2048	N/A	N/A
41.025	14.9831	42.678	9.76057	41.025	17.5822

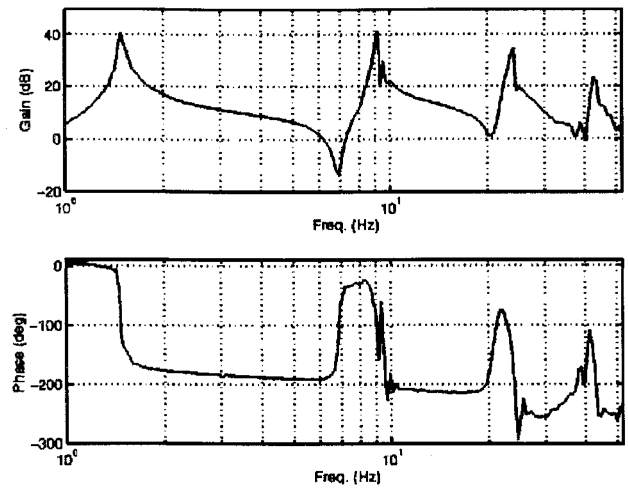

Fig. 3 Experimental Bode response: one sensor at station 20.

of working locations for identification of the structure's modes is very important for ground testing information because mode shapes are less sensitive to change from ground to space environments.[1] Information from ground experiments can be used to decide where the sensors should be located and also to decide where possible backups can be placed.

The sensor placement algorithm was implemented and written using the MATLAB[5] software package. Although the algorithm was validated experimentally, it is not immune to its own flaws. Clearly, a disadvantage is its exhaustive and combinatorial routines that increases the computational cost factorially. The 21 candidate sensor locations required a total of $2^{10} - 1$ possible combinations. This search is computationally intensive both in CPU and disk space requirements. Imagine a mode shape vector defined by 100 candidate sensor locations. The required combination is on the order of 10^{30}. This problem can be bypassed, however, by initially reducing the original set of candidate locations to a value that can be computationally implemented. This requires some educated guessing and insight on the part of the researcher. Certainly, there are locations where it is intuitively obvious that sensors are needed and where sensors are not needed. Another solution is to use information on the number of actual physical sensors that is available for experimental use and include this information in the combinatorial process in the algorithm. All of the possible combinations computed will be based on this number. For example, if only 10 physical sensors are available for the 100 candidate sensor locations, the algorithm will take into account that only 10 sensor's locations are available and use this information during the computation process. The sweeping process will only find the optimal locations for 1–10 sensors from the 100 possible candidate locations. A third solution is to look at each significant section of the structure and solve the sensor placement problem individually (i.e., long truss boom, dish radar/antenna, etc.).

Conclusion

An experimental evaluation of a predeveloped sensor placement algorithm is presented. The approach taken for the optimal sensor placement was based on a result from the D-optimal experiment design for identification. The D-optimality criterion, defined as the maximum determinant of the Fisher information matrix, results in a separation principle that decouples the problems of optimal input design and optimal sensor design such that each can be solved independently of each other. The sensor placement algorithm utilized in this study determines the optimal number of sensors needed to identify and detect all possible motions. In addition, for each number of sensors used, the algorithm also provides the best location for the sensor(s). The optimal sensor placement design was experimentally verified on a 63-degree-of-freedom planar truss, and the results were presented. It was shown how the top five optimal locations for just one sensor were able to identify all four of the significant natural frequencies of the planar truss. This information is useful for ground testing and design in deciding where the sensors should be located and where possible backups can be placed.

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Novel Approach to Reconfigurable Control Systems Design

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Introduction

RECONFIGURABLE control systems (RCS) are control systems possessing the ability to accommodate system failures automatically based on a priori assumed conditions. The research in this area is motivated largely by problems encountered in flight control systems. The goal here is to achieve the fault tolerant capability so that unanticipated failures in the system can be accommodated and the airplane at least can be landed safely whenever possible. Because of the time constraint, the control law redesign process has to be automated, and the algorithms should be numerically efficient.

Clearly, reconfigurable control is a very challenging control design problem. It requires that the failure detection and identification (FDI), the parameter estimation, and controller redesign be carried out on-line and, in many cases, completed in a short time span. Such a task is enormously complex, and it poses many interesting topics for research. In this Note, we address the controller redesign problem. Note that most standard control design methodologies cannot be directly applied here because they can only be used off-line, and the design procedure is usually iterative and time consuming. Our objective here is to develop a design method as simple and efficient as possible while providing a certain performance guarantee.

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